

# Unit 13 Fractions

## Fractions and mixed numbers

### Remember

A fraction has two parts.

The **denominator** tells you the number of equal parts the whole is divided into.

$$\frac{2}{3}$$

The **numerator** tells you the number of those equal parts that are taken.

If the numerator is smaller than the denominator, it is a **proper fraction**.

For example:  $\frac{3}{5}$

If the numerator is larger than the denominator, it is an **improper fraction**. It can also be written as a **mixed number**.

For example:  $\frac{7}{3}$  or  $2\frac{1}{3}$

### Changing mixed numbers to improper fractions

Multiply the denominator by the whole number and then add the numerator.

#### Example 1

$$2\frac{3}{4} \rightarrow \square$$

$$2 \times 4 + 3 = 11$$

$$2\frac{3}{4} = \frac{11}{4}$$

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### Changing improper fractions to mixed numbers

Divide the numerator by the denominator to find how many whole numbers there are and any fractions remaining.

#### Example 2

$$\frac{13}{5} \rightarrow \square$$

$$\frac{13}{5} = 13 \div 5 = 2 \text{ r } 3$$

$$\frac{13}{5} = 2\frac{3}{5}$$

1 Copy and complete these.

a)  $2\frac{1}{4} = \frac{\square}{4}$

b)  $3\frac{2}{3} = \frac{\square}{3}$

c)  $1\frac{7}{8} = \frac{\square}{8}$

d)  $2\frac{3}{10} = \frac{\square}{10}$

e)  $4\frac{1}{2} = \frac{\square}{2}$

f)  $1\frac{3}{4} = \frac{\square}{4}$

g)  $3\frac{7}{8} = \frac{\square}{8}$

h)  $4\frac{1}{3} = \frac{\square}{3}$

**2** Change these to mixed numbers or whole numbers.

a)  $\frac{9}{4}$

b)  $\frac{17}{3}$

c)  $\frac{7}{6}$

d)  $\frac{15}{5}$

e)  $\frac{19}{2}$

f)  $\frac{22}{8}$

**3** Change these to improper fractions.

a)  $3\frac{1}{5}$

b)  $2\frac{3}{8}$

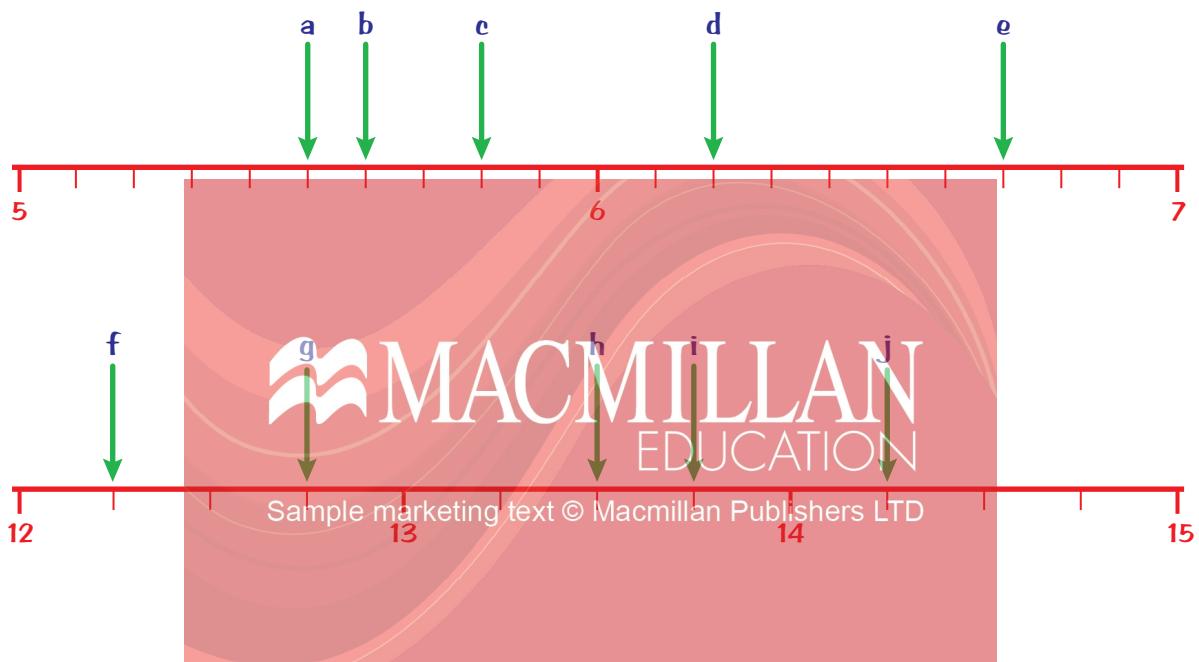
c)  $4\frac{2}{3}$

d)  $5\frac{1}{2}$

e)  $8\frac{7}{10}$

f)  $6\frac{3}{4}$

**4** Write the fraction each arrow points to, as an improper fraction and a mixed number.



**Try this**

Use these numbers.

2

7

3

11

5

- a) Use pairs of the numbers to make fractions less than 1.
- b) Use pairs of the numbers to make fractions greater than 1.

# Equivalent fractions

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} \dots \text{ and so on}$$

These fractions are all equivalent. They are worth the same.

To change a fraction to an equivalent fraction, we multiply the numerator and the denominator by the same number.

## Example 1

$$\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10}$$

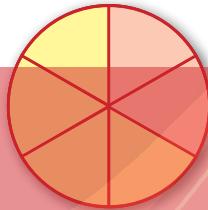
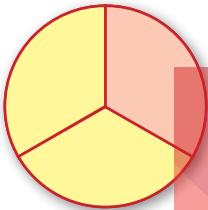
To reduce a fraction to its simplest form, we divide the numerator and the denominator by the same number.

## Example 2

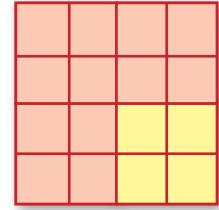
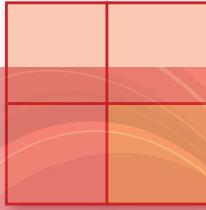
$$\frac{4}{16} = \frac{4 \div 4}{16 \div 4} = \frac{1}{4}$$

**1** Write the pairs of equivalent fractions for the pink part of each shape.

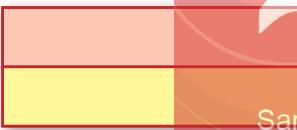
a)



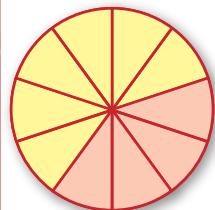
b)



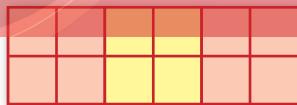
c)



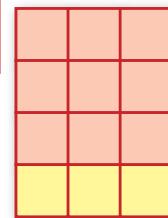
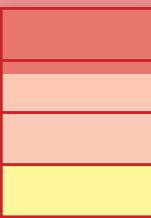
d)



e)



f)



**2** Complete the pairs of equivalent fractions.

a)  $\frac{2}{3} = \frac{\square}{6}$

b)  $\frac{5}{6} = \frac{10}{\square}$

c)  $\frac{\square}{2} = \frac{5}{10}$

d)  $\frac{1}{\square} = \frac{3}{9}$

e)  $\frac{\square}{6} = \frac{30}{36}$

f)  $\frac{3}{5} = \frac{\square}{20}$

g)  $\frac{1}{3} = \frac{6}{\square}$

h)  $\frac{7}{\square} = \frac{35}{50}$

i)  $\frac{3}{13} = \frac{\square}{39}$

j)  $\frac{7}{100} = \frac{56}{\square}$

k)  $\frac{\square}{9} = \frac{6}{27}$

l)  $\frac{6}{\square} = \frac{36}{42}$

**3** Copy and complete these.  
Write each fraction in its simplest form.

**a)**  $\frac{2}{4} \xrightarrow{\div 2} \frac{?}{?}$   
 $\frac{2}{4} = \frac{?}{?}$   
 $\xrightarrow{\div 2}$

**b)**  $\frac{6}{10} \xrightarrow{\div 2} \frac{?}{?}$   
 $\frac{6}{10} = \frac{?}{?}$   
 $\xrightarrow{\div 2}$

**c)**  $\frac{9}{12} \xrightarrow{\div 3} \frac{?}{?}$   
 $\frac{9}{12} = \frac{?}{?}$   
 $\xrightarrow{\div ?}$

**d)**  $\frac{5}{10} \xrightarrow{\div 5} \frac{?}{?}$   
 $\frac{5}{10} = \frac{?}{?}$   
 $\xrightarrow{\div 5}$

**e)**  $\frac{6}{8} \xrightarrow{\div 2} \frac{?}{?}$   
 $\frac{6}{8} = \frac{?}{?}$   
 $\xrightarrow{\div ?}$

**f)**  $\frac{15}{20} \xrightarrow{\div 5} \frac{?}{?}$   
 $\frac{15}{20} = \frac{?}{?}$   
 $\xrightarrow{\div ?}$

**4** Reduce these fractions to their simplest form.

**a)**  $\frac{2}{4}$

**b)**  $\frac{12}{14}$

**c)**  $\frac{5}{20}$

**d)**  $\frac{7}{28}$

**e)**  $\frac{8}{8}$

**f)**  $\frac{6}{15}$

**g)**  $\frac{10}{25}$

**h)**  $\frac{6}{16}$

**i)**  $\frac{20}{28}$

**j)**  $\frac{15}{24}$

**k)**  $\frac{28}{63}$

**l)**  $\frac{24}{64}$



**Try this**

Look at this fraction  $\frac{19}{95}$ .

If you cross out the two digits that are the same, you make an equivalent fraction.

$$\frac{19}{95} = \frac{1}{5}$$

Find other pairs of fractions where this is true.

# Comparing and ordering fractions

It is easier to compare fractions if they have the same denominator. If the denominators are not the same, we need to change them to equivalent fractions with a **common denominator**. To do this, you need to know the **lowest common multiple** for each denominator.

## Example

Which is the larger fraction,  $\frac{3}{5}$  or  $\frac{3}{4}$ ?

First, change them to equivalent fractions.

The lowest common multiple of 5 and 4 is 20.

$$\frac{3}{5} = \frac{3 \times 4}{5 \times 4} = \frac{12}{20}$$

$$\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$$

$$\frac{15}{20} > \frac{12}{20} \text{ so } \frac{3}{4} \text{ is greater than } \frac{3}{5}$$

- 1** Write  $<$ ,  $>$  or  $=$  between each pair of fractions. Remember to change them to equivalent fractions.

a)  $\frac{2}{3}$    $\frac{5}{6}$

b)  $\frac{4}{7}$    $\frac{3}{4}$

c)  $\frac{1}{3}$    $\frac{2}{5}$

d)  $\frac{3}{8}$    $\frac{2}{7}$

e)  $\frac{4}{5}$    $\frac{7}{10}$

f)  $\frac{5}{8}$    $\frac{3}{10}$

g)  $\frac{7}{12}$    $\frac{3}{4}$

h)  $\frac{4}{5}$    $\frac{17}{20}$

- 2** Find the common denominators for these fractions and write them in order, starting with the smallest. Then write the original fractions in order.

a)  $\frac{2}{7}$

$\frac{4}{5}$

$\frac{1}{2}$

b)  $\frac{6}{15}$

$\frac{2}{3}$

$\frac{3}{10}$

c)  $\frac{3}{8}$

$\frac{7}{10}$

$\frac{1}{4}$

d)  $\frac{3}{5}$

$\frac{9}{12}$

$\frac{8}{15}$

e)  $\frac{2}{3}$

$\frac{19}{20}$

$\frac{2}{5}$

f)  $\frac{3}{10}$

$\frac{6}{14}$

$\frac{2}{7}$

g)  $\frac{5}{16}$

$\frac{2}{5}$

$\frac{7}{20}$

h)  $\frac{5}{21}$

$\frac{1}{3}$

$\frac{2}{9}$

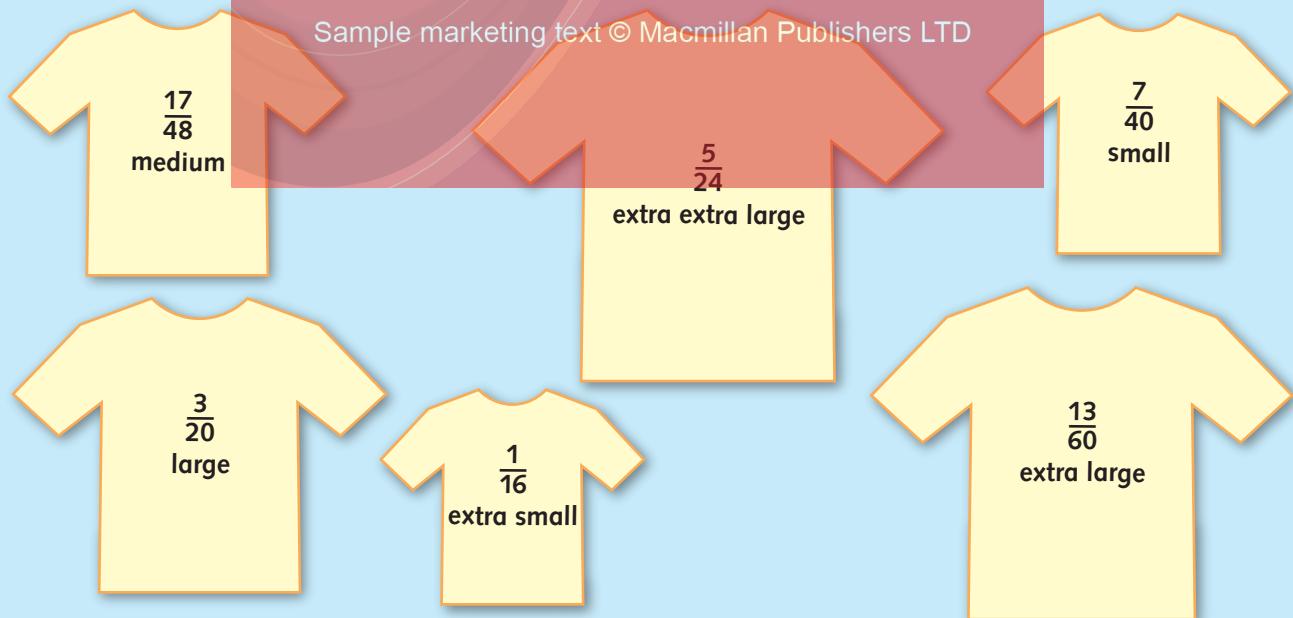
**3** Answer these problems.

- a) A market stall has the same number of oranges and lemons to sell. After an hour  $\frac{5}{8}$  of his oranges are sold and  $\frac{7}{12}$  of his lemons are sold. Which fruit has sold the most?
- b) Alex and Sophie each have a bag of sweets with the same number of sweets in each bag. Alex has  $\frac{7}{12}$  of his sweets left and Sophie has  $\frac{5}{6}$  of her sweets left. Who has eaten the most sweets?
- c) When Newtown played Oldcastle, the stadium was  $\frac{7}{12}$  full. When they played Summertown later in the season the stadium was  $\frac{3}{5}$  full. Which match had the biggest crowd?
- d) Harry had a long car journey to make over three days. He drove  $\frac{3}{8}$  of the distance on the first day and  $\frac{5}{12}$  of the distance on the second day. On which of these two days did he drive the furthest?
- e) In a library there are fiction, non-fiction and poetry books.  $\frac{8}{15}$  of the books are non-fiction and  $\frac{5}{12}$  of them are fiction. Are there more fiction or non-fiction books?

**Try this**

Read this and answer the questions below.

T-shirts are sold in a store. These are the sizes of the t-shirts. The amount of each size is shown as a fraction.



- a) Which size t-shirt are there most of?
- b) Which size t-shirt are there fewest of?
- c) Are there fewer large t-shirts or small t-shirts?
- d) Are there more extra-large t-shirts or extra-extra-large t-shirts?
- e) There are 240 t-shirts in the store. How many of each size are there?

# Adding and subtracting fractions

Fractions with different denominators are unlike fractions.

To add or subtract unlike fractions, we change them to like fractions by looking for equivalent fractions with a common denominator.

Follow these steps.

**Step 1** Find equivalent fractions with a common denominator.

**Step 2** Add the numerators and write the numerator over the common denominator.

**Step 3** Simplify the fraction if needed.

## Example 1

Add  $\frac{3}{4}$  and  $1\frac{1}{5}$

Common denominator is 20.

$$\frac{15}{20} + 1\frac{4}{20} = 1\frac{19}{20}$$

## Example 2

Subtract  $\frac{2}{5}$  from  $\frac{2}{3}$

Common denominator is 15.

$$\frac{10}{15} - \frac{6}{15} = \frac{4}{15}$$

- 1** Find the common denominator for the fractions in each addition. Write the answers in their simplest form.

a)  $\frac{7}{8} + \frac{3}{4} =$

b)  $\frac{4}{5} + \frac{3}{10} =$

c)  $\frac{2}{3} + \frac{11}{15} =$

d)  $\frac{7}{10} + \frac{3}{8} =$

e)  $\frac{3}{4} + \frac{2}{7} =$

f)  $\frac{3}{5} + \frac{11}{12} =$

g)  $6\frac{2}{7} + 3\frac{1}{3} =$

h)  $4\frac{7}{10} + 5\frac{7}{8} =$

i)  $2\frac{1}{2} + 6\frac{3}{11} =$

j)  $4\frac{11}{12} + 1\frac{2}{3} =$

k)  $5\frac{5}{6} + 4\frac{3}{8} =$

- 2** Find a common denominator and subtract these. Simplify the answer if needed.

a)  $\frac{4}{5} - \frac{3}{10} =$

b)  $\frac{2}{3} - \frac{5}{9} =$

c)  $\frac{4}{7} - \frac{3}{10} =$

d)  $\frac{7}{8} - \frac{1}{3} =$

e)  $\frac{4}{5} - \frac{3}{20} =$

f)  $\frac{7}{10} - \frac{4}{15} =$

g)  $2\frac{1}{6} - \frac{3}{8} =$

h)  $1\frac{3}{5} - \frac{2}{9} =$

i)  $2\frac{11}{12} - \frac{3}{4} =$

j)  $4\frac{6}{7} - 1\frac{1}{3} =$

k)  $5\frac{3}{8} - 2\frac{4}{5} =$

l)  $3\frac{7}{20} - 1\frac{2}{3} =$

**3** Add together each group of fractions.

- a)  $\frac{5}{6}$  +  $\frac{2}{3}$  +  $\frac{1}{12}$
- b)  $\frac{3}{10}$  +  $\frac{2}{5}$  +  $\frac{11}{15}$
- c)  $2\frac{1}{2}$  +  $1\frac{3}{4}$  +  $2\frac{3}{5}$
- d)  $4\frac{3}{10}$  +  $5\frac{2}{3}$  +  $1\frac{5}{6}$

**4** Read and answer these.

- a) What is  $6\frac{3}{8}$  less than  $9\frac{4}{5}$ ?
- b) Subtract  $3\frac{7}{10}$  from  $6\frac{7}{15}$ .
- c) What is the difference between  $12\frac{3}{5}$  and  $8\frac{7}{8}$ ?
- d) What is  $12\frac{7}{18}$  take away  $9\frac{11}{12}$ ?
- e) What is  $11\frac{7}{20}$  subtract  $8\frac{1}{12}$ ?
- f) What is the difference between  $15\frac{9}{25}$  and  $12\frac{9}{10}$ ?

### Try this

Copy and complete these.  
Use each of the digits 1 to 6 to fill in the boxes.

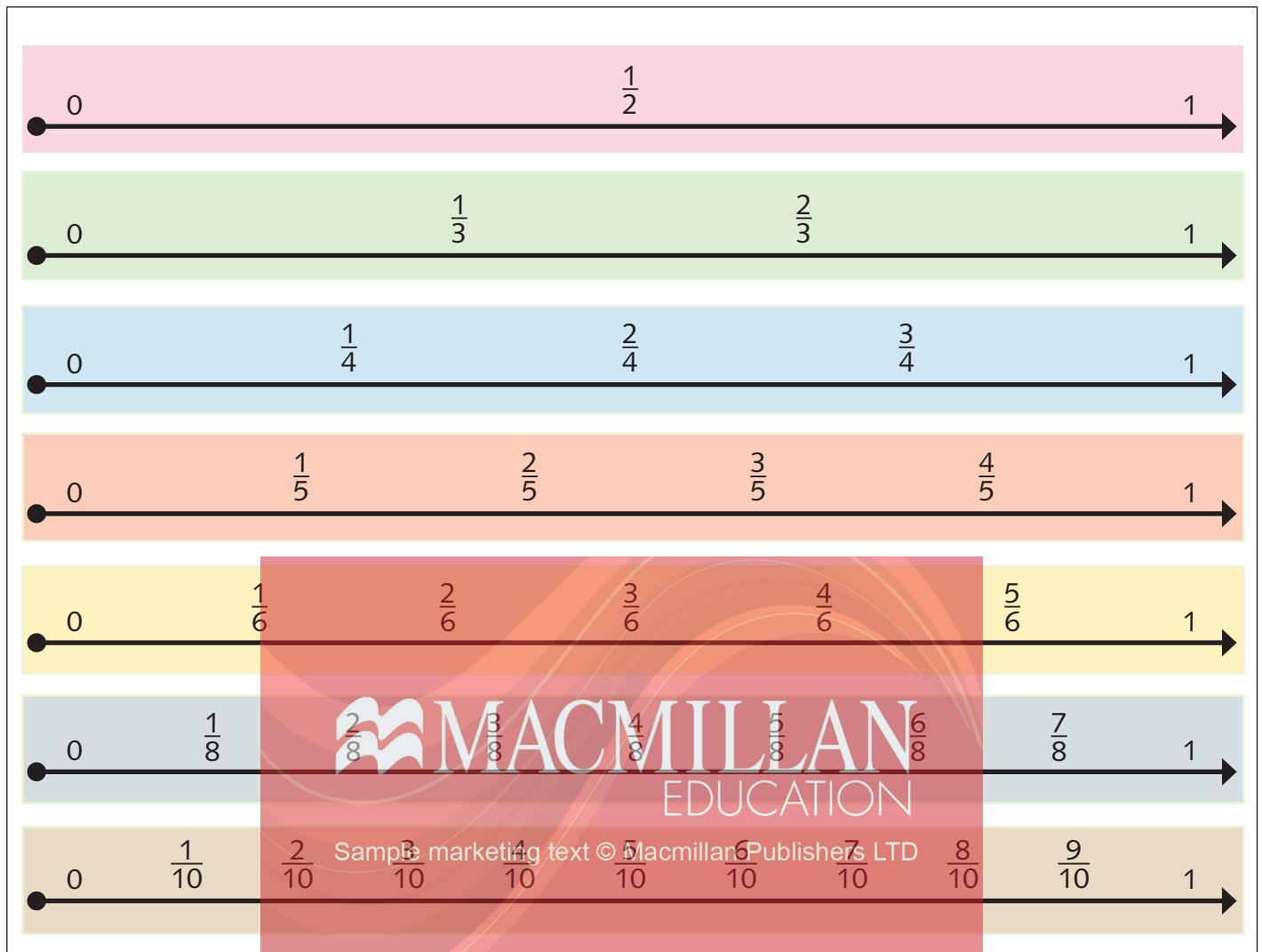


$$\frac{\square}{10} + \frac{\square}{5} = \frac{7}{10}$$

$$\frac{1}{\square} + \frac{1}{3} = \frac{\square}{2}$$

$$\frac{3}{8} + \frac{1}{\square} = \frac{\square}{8}$$

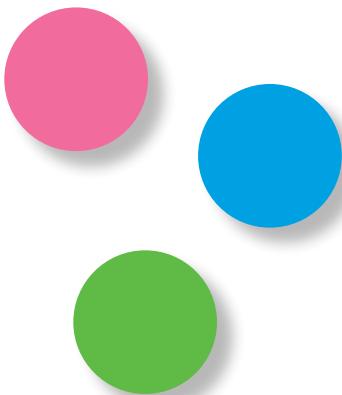
# Fraction game



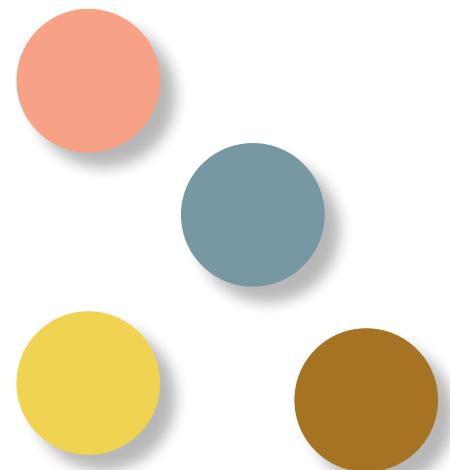
Play **'Fraction add'** on this game board.

Each player needs a set of 7 coloured counters.

Copy each of these fractions onto small pieces of paper. Cut them out and put them in a bag or box.



$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{3}{4}$
$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{1}{6}$
$\frac{5}{6}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$
$\frac{1}{10}$	$\frac{3}{10}$	$\frac{7}{10}$	$\frac{9}{10}$	1



## How to play

- Each player places a counter at 0 on each line. (This uses all their counters.)
- The first player, without looking, picks out a paper fraction from the bag or box. The player moves one or more of their counters to the right along the number lines to total the fraction picked out. For example, if  $\frac{3}{8}$  is picked out, two counters could be moved to  $\frac{1}{4}$  and  $\frac{1}{8}$ .
- After the total has been made and counters moved, the next player takes a turn.
- If the total cannot be made, then just pass on to the turn of the next player.
- Players return the paper fractions to the bag after each turn.

The winner is the first player to move all their counters to the 1 on the right of each number line.

## Assessment

**1** Look at the fractions and answer the questions.

- a) Which two fractions are equivalent?
- b) Which fraction in its simplest form is  $\frac{3}{20}$ ?
- c) Which fraction simplifies to  $\frac{7}{12}$ ?
- d) Which fraction is equivalent to  $\frac{1}{8}$ ?

**2** Look at the fractions and answer the questions.

- a) Make pairs of fractions from the set, so that you have three pairs.
- b) Write each fraction in its simplest form.
- c) Add each pair of fractions together. Simplify the answers.
- d) Write the three answers in order, starting with the smallest fraction.

$$\frac{56}{64}$$

$$\frac{22}{24}$$

$$\frac{35}{42}$$

$$\frac{42}{72}$$

$$\frac{15}{100}$$

$$\frac{40}{48}$$

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